

Preface

Aims and Scope

The aim of this book is to teach foundational topics in stochastic dynamics such as stability, ergodicity and dynamic programming, with applications from economics and finance. As we travel down this path, we will delve into a variety of related fields, including simulation and numerical methods, fixed point theory, stochastic process theory, function approximation, and coupling.

In writing the book I had two main goals. First, I wanted to show that sound understanding of relevant mathematical concepts leads to effective algorithms for solving real world problems. Second, I wanted the book to be enjoyable to read, with an emphasis on building intuition. Hence the material is driven by examples—I believe the fastest way to grasp a new concept is through studying examples—and makes extensive use of programming to illustrate ideas. Running simulations and computing equilibria helps bring abstract concepts to life.

The primary intended audience is advanced undergraduate and, especially, beginning graduate students in economics. However, the techniques discussed in the second half of the book add some shiny new toys to the standard tool kit used for economic modeling, and as such they should be of interest to advanced graduate students and researchers. The book is as self-contained as possible, given space constraints.

Part I of the book covers material that all well-rounded graduate students should know. The style is relatively mathematical, and those who find the going hard might start by working through the exercises in appendix A. Part II is significantly more challenging. In designing the text it was not my intention that all of those who read part I should go on to read part II. Rather, part II is written for researchers and graduate students with a particular interest in technical problems. Those who do read the majority of part II will gain a very strong understanding of infinite-horizon dynamic programming and (nonlinear) stochastic models.

How does this book differ from other textbooks? There are several books on computational macroeconomics and macrodynamics that treat related topics. In compar-

ison, this book is not specific to macroeconomics. It should be of interest to (at least some) people working in microeconomics, operations research, and finance. Second, computation and theory are tightly integrated. When numerical methods are discussed, I have tried to emphasize mathematical analysis of the algorithms. Readers will acquire a strong knowledge of the probabilistic and function-analytic framework that underlies proposed solutions.

Like any text containing a significant amount of mathematics, the notation piles up thick and fast. To aid readers I have worked hard to keep notation minimal and consistent. Uppercase symbols such as A and B usually refer to sets, while lowercase symbols such as x and y are elements of these sets. Functions use uppercase and lowercase symbols such as f , g , F , and G . Calligraphic letters such as \mathcal{A} and \mathcal{B} represent sets of sets or, occasionally, sets of functions. Proofs end with the symbol \square .

I provide a table of common symbols on page xvii. Furthermore, the index begins with an extensive list of symbols, along with the number of the page on which they are defined.

Code and Online Resources

Code and online resources for the textbook can be found at

<https://johnstachurski.net/edtc.html>

On that page you will find a link to an online code book that accompanies this text, created using Jupyter Book. The code book contains Python code that generates the figures and runs computations discussed in the textbook. Solutions to exercises involving computation are also included.¹

Additional related code can be found at <https://quantecon.org>. The code there includes Python and Julia implementations of algorithms discussed in this text, such as routines for simulation of Markov chains and solution of Markov decision processes.

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