

Contents

Preface	xv
Common Symbols	xvii
1 Introduction	1
1.1 The Nature of Econometrics	1
1.2 Data versus Theory	3
1.3 Comments on the Literature	5
1.4 Further Reading	5
I Background	7
2 Vector Spaces	9
2.1 Vectors and Vector Space	9
2.1.1 Vectors	9
2.1.2 Linear Combinations and Span	14
2.1.3 Linear Independence	17
2.1.4 Linear Subspaces	20
2.1.5 Bases and Dimension	21
2.1.6 Linear Maps	23
2.1.7 Linear Independence and Bijections	25
2.2 Orthogonality	27
2.2.1 Definition and Basic Properties	27
2.2.2 The Orthogonal Projection Theorem	29
2.2.3 Projection as a Mapping	30
2.2.4 The Residual Projection	32
2.3 Further Reading	34
2.4 Exercises	35

2.4.1	Solutions to Selected Exercises	37
3	Linear Algebra and Matrices	45
3.1	Matrices and Linear Equations	45
3.1.1	Basic Definitions	45
3.1.2	Matrices as Maps	48
3.1.3	Square Matrices and Invertibility	50
3.1.4	Determinants	52
3.2	Properties of Matrices	53
3.2.1	Diagonal and Triangular Matrices	53
3.2.2	Trace, Transpose, and Symmetry	54
3.2.3	Eigenvalues and Eigenvectors	55
3.2.4	Quadratic Forms	57
3.3	Projection and Decomposition	60
3.3.1	Projection Matrices	60
3.3.2	Overdetermined Systems of Equations	62
3.3.3	QR Decomposition	64
3.3.4	Diagonalization and Spectral Theory	65
3.3.5	Norms and Continuity	67
3.4	Further Reading	70
3.5	Exercises	70
3.5.1	Solutions to Selected Exercises	73
4	Foundations of Probability	79
4.1	Probabilistic Models	79
4.1.1	Sample Spaces and Events	79
4.1.2	Probabilities	83
4.1.3	Random Variables	89
4.1.4	Expectations	93
4.1.5	Moments and Co-Moments	97
4.2	Distributions	99
4.2.1	Defining Distributions on \mathbb{R}	100
4.2.2	Densities and PMFs	103
4.2.3	Integrating with Distributions	108
4.2.4	Distributions of Random Variables	110
4.2.5	Expectations from Distributions	112
4.2.6	Quantile Functions	113
4.3	Further Reading	116
4.4	Exercises	116
4.4.1	Solutions to Selected Exercises	118

5	Modeling Dependence	125
5.1	Random Vectors and Matrices	125
5.1.1	Random Vectors	125
5.1.2	Multivariate Distributions	127
5.1.3	Distributions of Random Vectors	132
5.1.4	Independence	135
5.1.5	Copulas	138
5.1.6	Properties of Named Distributions	140
5.2	Conditioning and Expectation	141
5.2.1	Conditional Distributions	141
5.2.2	The Space L_2	142
5.2.3	Projections in L_2	145
5.2.4	Measurability	148
5.2.5	Conditional Expectation	150
5.2.6	The Vector Case	153
5.3	Further Reading	154
5.4	Exercises	154
5.4.1	Solutions to Selected Exercises	156
6	Asymptotics	161
6.1	LLN and CLT	161
6.1.1	Convergence of Random Vectors	161
6.1.2	The Law of Large Numbers	163
6.1.3	Convergence in Distribution	165
6.1.4	The Central Limit Theorem	168
6.2	Extensions	169
6.2.1	Convergence of Random Matrices	170
6.2.2	Vector-Valued LLNs and CLTs	171
6.2.3	The Delta Method	173
6.3	Further Reading	174
6.4	Exercises	174
6.4.1	Solutions to Selected Exercises	175
7	Further Topics in Probability	177
7.1	Stochastic Processes	177
7.1.1	Stationarity and Ergodicity	178
7.1.2	Stochastic Recursive Sequences	179
7.2	Markov Processes	184
7.2.1	The Markov Assumption	184
7.2.2	Marginal and Joint Distributions	188

7.2.3	Stationarity of Markov Processes	190
7.2.4	Asymptotics of Markov Processes	193
7.2.5	The Linear Case	196
7.3	Martingales	197
7.3.1	Definitions	197
7.3.2	Martingale Difference LLN and CLT	199
7.4	Simulation	200
7.4.1	Inverse Transforms	200
7.4.2	Markov Chain Monte Carlo	201
7.5	Further Reading	206
7.6	Exercises	206
7.6.1	Solutions to Selected Exercises	207
 II Foundations of Statistics		 211
8	Estimators	213
8.1	The Estimation Problem	213
8.1.1	Definitions	213
8.1.2	Statistics and Estimators	216
8.1.3	Empirical Distributions	219
8.2	Estimation Principles	222
8.2.1	The Sample Analogue Principle	222
8.2.2	Empirical Risk Minimization	225
8.2.3	The Choice of Hypothesis Space	228
8.3	Some Parametric Methods	233
8.3.1	Maximum Likelihood	234
8.3.2	Maximum Likelihood via ERM	238
8.3.3	The Method of Moments and GMM	239
8.3.4	Bayesian Estimation	241
8.4	Further Reading	244
8.5	Exercises	244
8.5.1	Solutions to Selected Exercises	245
9	Properties of Estimators	247
9.1	Sampling Distributions	247
9.1.1	Estimators as Random Elements	247
9.1.2	Sampling Distributions	248
9.1.3	The Bootstrap	251
9.2	Evaluating Estimators	255

9.2.1	Bias	256
9.2.2	Variance	257
9.2.3	Variance versus Bias	259
9.2.4	Asymptotic Properties	262
9.2.5	Decision Theory	265
9.3	Further Reading	270
9.4	Exercises	270
9.4.1	Solutions to Selected Exercises	272
10	Confidence Intervals and Tests	275
10.1	Confidence Sets	275
10.1.1	Finite Sample Confidence Sets	276
10.1.2	Asymptotic Methods	277
10.1.3	A Nonparametric Example	279
10.2	Hypothesis Tests	280
10.2.1	Constructing Tests	282
10.2.2	Choosing Critical Values	284
10.2.3	Asymptotic Tests	286
10.2.4	Accepting the Null?	289
10.2.5	Statistical Tests in Economics	293
10.3	Further Reading	294
10.4	Exercises	295
10.4.1	Solutions to Selected Exercises	296
III	Econometric Models	297
11	Regression	299
11.1	Linear Regression	299
11.1.1	The Setup	299
11.1.2	The Least Squares Estimator	301
11.1.3	Out-of-Sample Fit	304
11.1.4	In-Sample Fit	306
11.2	The Geometry of Least Squares	308
11.2.1	Transformations and Basis Functions	308
11.2.2	The Frisch–Waugh–Lovell Theorem	311
11.2.3	Centered Observations	314
11.3	Further Reading	315
11.4	Exercises	315
11.4.1	Solutions to Selected Exercises	317

12 Ordinary Least Squares	323
12.1 Estimation under OLS	323
12.1.1 Assumptions	323
12.1.2 The OLS Estimators	325
12.1.3 Finite Sample Properties	327
12.1.4 Inference with Normal Errors	331
12.2 Problems and Extensions	338
12.2.1 Nonspherical Errors	338
12.2.2 Bias	340
12.2.3 Instrumental Variables	343
12.2.4 Causality	345
12.3 Further Reading	347
12.4 Exercises	347
12.4.1 Solutions to Selected Exercises	349
13 Large Samples and Dependence	355
13.1 Large Sample Least Squares	355
13.1.1 Setup and Assumptions	355
13.1.2 Consistency	358
13.1.3 Asymptotic Normality of $\hat{\beta}$	359
13.1.4 Large Sample Tests	361
13.2 MLE for Markov Processes	363
13.2.1 The Likelihood Function	363
13.2.2 The Newton–Raphson Algorithm	365
13.3 Further Reading	370
13.4 Exercises	370
13.4.1 Solutions to Selected Exercises	372
14 Regularization	377
14.1 Nonparametric Density Estimation	377
14.1.1 Introduction	377
14.1.2 Kernel Density Estimation	379
14.1.3 Theory	381
14.1.4 Commentary	386
14.2 Controlling Complexity	386
14.2.1 Ridge Regression	387
14.2.2 Subset Selection and Ridge Regression	389
14.2.3 Bayesian Methods and Regularization	392
14.2.4 Cross-Validation	395
14.3 Further Reading	399

Contents	xiii
14.4 Exercises	399
14.4.1 Solutions to Selected Exercises	400
IV Appendix	403
15 Appendix	405
15.1 Sets	405
15.1.1 Cartesian Products	408
15.2 Functions	408
15.2.1 Preimage of Sets	411
15.3 Cardinality and Measure	411
15.3.1 Lebesgue Measure and Sets of Measure Zero	412
15.4 Real-Valued Functions	413
15.4.1 Sup and Inf	413
Bibliography	415
Index	425